Ergodicity and Investments

Part 1

by John Schuyler

Everything We've Learned about Modern Economic Theory Is Wrong

"Ole Peters, a theoretical physicist in the U.K., claims to have the solution. **All it would do is upend three centuries of economic thought.**"

by Brandon Kochkodin, posted to *Bloomberg News*, December 11, 2020.

https://www.bloomberg.com/news/articles/2020-12-11/everything-we-ve-learned-about-moderneconomic-theory-is-wrong (now behind the Bloomberg paywall)

(Additional references are at the end of Part 2.)

A friend and colleague, Dr. James Applegate, sent me the link to this article. It sent me on a journey to investigate and learn about ergodicity. Is there something wrong with *Expected Monetary Value* (*EMV*) or *Expected Utility* decision policy?

From Wikipedia:

<u>Ergodicity</u> is a property of the system; it is a statement that we cannot reduce the system or factor it into smaller components.

Ergodic theory is the study of systems possessing ergodicity. Ergodic systems occur in a broad range of systems in physics and in geometry.



Admiral Haney flips the coin at the Pro Bowl Source: Wikipedia Commons

Ergodicity appears to have important applications to economics. Ole Peters demonstrates that some typical investment strategies vastly underperform expected value forecasts.

Consider this experiment: You flip a fair coin 100 times and record the sequence of heads and tails. There are 2¹⁰⁰ possible sequences. If you keep repeating the experiment, you will eventually realize all possible sequences. This dispersion is ergodicity. Many physical systems exhibit this behavior. As examples:

- Smoke introduced into a room eventually disperses throughout the room.
- Heat temporarily applied to an area of a metal plate redistributes to all areas of the plate.

Take some time to consider this investment:

Flipping a fair coin has equal chances of landing on either side and zero chance of staying on edge. Commonly, one side has an image of a famous person, we call this side "Heads" or H. The other (obverse) side is "Tails" or T.

Flipping the coin determines your outcome for the bet. You can bet (flip) as many times as desired and stop at any time.

Suppose you start with a \$100 bet. You can assume the initial bet is \$100k if that makes it more interesting. You may prefer to consider this a risky "investment" rather than a gamble.

After a first flip, your value either gains 50% (Heads) or loses 40% (Tails). So, you have either \$150 or \$60 to apply to the next flip.

The expected monetary value (EMV, the probability-weighted outcome) of the initial flip is:

EMV = 0.5 × (\$100 × 0.5) + 0.5 × (-\$100 × 0.4) = \$5

This is the expected value change amount. So, on average, you would win \$5 for a \$100, one flip, gamble.

You may continue for as long as desired. At every flip, you either gain 50% or lose 40%.

Example: Consider the four possible outcomes of the first two flips.

After two flips, your investment is worth one of these equally probable outcomes:

| HH | \$100 x 1.5 x 1.5 = | \$225 |
|---------------------------------|---------------------|-------------|
| ΗT | \$100 x 1.5 x 0.6 = | \$90 |
| ΤH | \$100 x 0.6 x 1.5 = | \$90 |
| TT | \$100 x 0.6 x 0.6 = | <u>\$36</u> |
| <i>EMV</i> = average = \$110.25 | | |

Questions:

- a. Would you invest?
- b. If so, how much would you invest? More or less than \$100 for each flip? Express your answer in \$ or as a fraction of your net worth.
- c. Would it matter if you could repeat the investment as many times as desired? Or fewer times? How would you play?

Please consider these questions carefully before continuing to Part 2.

Originally posted May 31, 2021; revised November 24, 2023

Disclaimer: This note is not investment advice. Though the concept is general, it may not apply to your circumstance.

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