2. EXPECTED VALUE

Uncertainty and ***risk*** are used to characterize unknowns. In some situations, such as well-known games of chance, we can establish the exact probabilities for all possible results. In other situations, such as with nature, we may never know the exact odds. Nonetheless, we may still assign probabilities to our expectations based upon experience and intuition.

People disagree on definitions for *risk* and *uncertainty*.

Risk is a term often used interchangeably with uncertainty. Where uncertainty often refers the expectation of values within a range, risk is often reserved for more dramatic differences between possible outcomes. For example, one might refer to *uncertainty* of a range of prices and *risk* of an exploration or research project's success. Risk is also used commonly to refer to a severe potential downside.

Expected Value (EV) is a measure of our expectations about the outcome of a chance event. An example illustrates the concept.

> Consider a simple coin toss lottery. If heads, your payoff is $2; if tails, you win only $1. Suppose you must decide how much to pay for the opportunity to play. A fair coin would provide a 50:50 chance of winning either $1 or $2. One would expect, then, the value of a chance to play to be somewhere between $1 and $2.

If many of these opportunities were available, you would expect to win $2 half of the time and $1 half of the time. So, for each time you played the coin toss you would expect to receive a $1.50 <u>on average</u>. $1.50 is the ***expected value*** (EV) payoff for each trial of the game, not including the amount you pay to play. When the results are expressed in monetary amounts, the expected value is called expected monetary value (EMV).[1]

probability-weighted average

Expected value is the probability-weighted average of all possible outcomes of a chance event:

$$EV = Result_1 P_1 + Result_2 P_2 + ... + Result_n P_n$$

Where $Result_i$ is the value of the i^{th} outcome

P_i is the probability of result i

The n results should represent all possible outcomes so that the probabilities sum to 1, that is $\sum_{\text{all } i} p_i = 1$.

[1] Especially when the expectation is positive. If only costs are involved, I prefer the label “expected value cost.”

Example

Consider the product development evaluation shown by the table. The ***chance event*** under consideration is the sales level that will be realized. For simplicity, the sales level is characterized as having only 3 possible outcomes: high, medium, or low. The net monetary result of each potential sales level, the outcome of this chance event, is expressed as a present value.

EXPECTED VALUE CALCULATION
Amounts in 000s

SALES LEVEL	PV VALUE OF OUTCOME	X	PROBABILITY OF OUTCOME	=	EXPECTED MONETARY VALUE
HIGH	$2000		.40		$800
MEDIUM	1000		.30		300
LOW	-500		.30		-150
			1.00		EMV = $950

A good decision rule for most situations is to consistently choose alternatives that have the highest EMVs. This strategy maximizes the EMV of the company.

If we could launch many products that all have similar potential, then we could expect to earn about $950,000 per product on average.

A ***decision rule*** is a procedure for selecting from among alternatives. Your best decision, all other factors being equal, is to choose the alternative with the highest expected value. For companies seeking to maximize their monetary value, the decision criterion should be EMV, that is, EV present value.

If you will have many similar decisions, your average result over the long run will be the expected value of the best alternative for a single decision. The EMV concept is most transparent if we have enough events so that statistics apply. However, most projects are unique—never to be repeated. Is EMV useful when we have only one decision to make? Yes if (a) your job (or other capacity) requires you to make a succession of decisions and (b) no one decision exposes you to unbearable risk. In a person's or business's life there are many decisions to be made, so the decision portfolio idea still applies.

Properties of EV

Expected value has these convenient, useful, and important properties:

Use EV with value measures.

- The delta property. Subtracting a cost, Δ, to every outcome decreases the EV by Δ..
- Multiplying every outcome by a factor α changes EV by a factor of α..

In decision policy, we need to ensure that the key decision criterion is a ***value measure***, such as reserves or money. Expected value works well with values, yet does poorly with non-value measures such as internal rate of return, payout, finding costs and other such criteria.

When Capital is Limited

Many people modify the EMV decision rule when there is limited capital or other constraining resource. This conflicts with a customary assumption in capital investment analysis: the cost of capital. We usually presume that we ***know*** (or can estimate) the cost of incremental capital (money) to fund that next project).

In situations where there is insufficient capital (or other resource), decision-makers desire to get the most value for each investment dollar. The portfolio of investment alternatives can be ranked according to the (risk-weighted) profitability index:

$$\text{PI} = \frac{\text{EMV}}{\text{EV investment}}$$

Most companies prefer to discount the denominator.[2] The ranking ratio is then (risk-weighted) ***discounted return on investment*** (DROI).[3] In ranking projects, companies should approve investments in order of decreasing DROI until the supply of capital is exhausted. There are optimization techniques that can deal with more complex “knapsack” problems, but PI or DROI is adequate for most situations. Be sure to examine any relationships, especially synergies that affect multiple candidate projects.

You may want to risk and/or discount the cost denominator depending on the capital budget period, how successful ventures are later-stage funded, and the payback characteristics of the investments.

EMV is the logical value measure when the objective is making money and the organization is neutral toward risk. Determining the appropriate discount rate is a subject of considerable interest and debate. We usually presume an infinite source and sink for funds at the discount rate. Then, the investments chosen with either PI or DROI are exactly the same as those selected with the EMV decision rule.

[2] I recommend not discounting the investment (hence PI instead of DROI), because I know of no company that discounts budget dollars. However, if the investment is made across several years, this strengthens the argument for using PV Investment. It’s an imperfect theory in an imperfect world.

[3] Another popular name for this is *Discounted Profit-to-investment Ratio* (DPR).