# 7. DECISION TREES

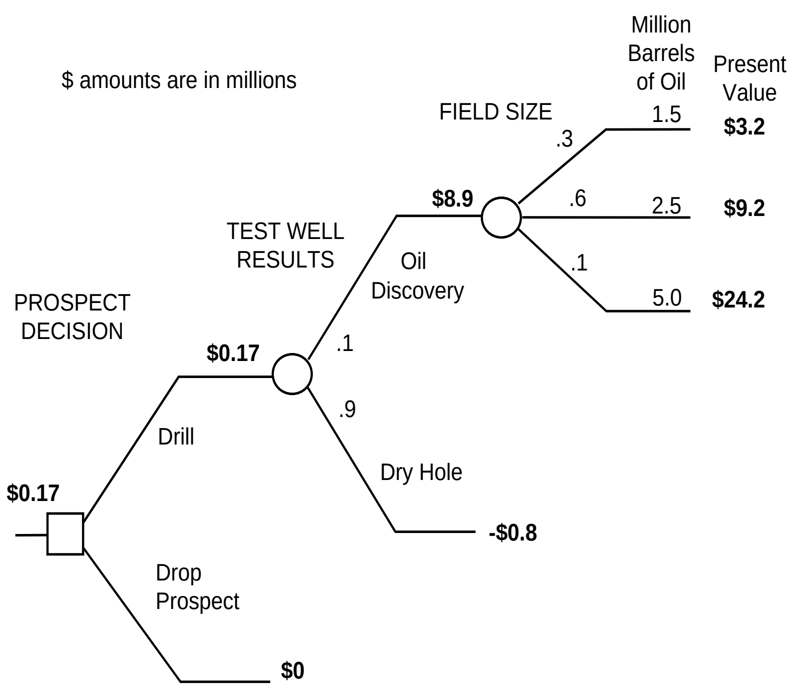
A decision tree is a convenient and powerful technique to visualize the logic of a decision problem and to compute expected values. A simple example is shown below. The tree consists of nodes and branches. There are three types of nodes:

- decision
- chance event
- terminal

The node sequence must match your problem.

By convention, decision nodes are drawn as boxes, and chance event nodes are drawn as circles. Terminal nodes are the branch endpoints, the "leaves," and the customary values are NPV outcomes.

The branches radiating from a decision node represent decision alternatives. The branches radiating from a chance event node represent possible outcomes. The ordering of nodes in the decision tree is very important. The nodes must be in the sequence in which information will become known and in which decisions will be made. By convention, time is represented in tree diagrams as progressing from left to right.



The decision modeled here is whether or not to drill an exploration test well. This will confirm (“prove”) or reject the geologic ideas supporting the prospect. If unsuccessful, the company bears $0.8 million dry hole cost. For success, possible field sizes and outcome dollar values are characterized by a 3-level distribution for reserves, in barrels of oil. $8.9 million is the expected monetary value of future net cash flow present values, given the test well results in a field discovery. $0.17 million is the expected monetary value of the "Drill" decision. , which is better than the $0 value of an "Drop Prospect" decision. No sunk costs, i.e., costs already incurred, should be represented in a decision tree’s values except to the extent that prior expenditures impact future cashflows due to taxes or contract terms.

Ignore sunk costs.

A complete decision tree represents all significant alternatives and the possible outcomes for each alternative. Often a chance event has a continuous range of possible values (e.g., size of an oil discovery). The most complete way to represent judgment about the chance event would be a continuous probability distribution. However, solvable decision trees must have a finite (and small) numbers of possible outcomes. So we must characterize continuous distributions with several representative values rather than the continuous distribution. As in the example, we commonly use a 3-level discrete distribution (high, medium, and low values) to approximate a continuous distribution in a decision tree.

Three-level discrete distributions

Similarly, some decision variables, such as a competitive bid amount, have a continuous range of alternatives. Decisions, too, must be characterized by several values.[1]

A decision tree is a decision model. In addition, the decision tree structure is useful in calculating the expected values of the alternatives. Outcome values for all paths through the tree are obtained from an underlying cashflow projection model. Value is usually expressed in either dollars or "utility" units. Then, the decision tree is folded back or back-solved to calculate expected values for each decision alternative. The alternative with the highest expected value is the best choice.[2] In the example tree, the expected values are shown in bold print above each node.

back-solving

The back-solving process is straightforward. You start at the endpoints on the right and work your way left:

- When you come to a chance node, calculates its expected value. This becomes the value of the node and the branch leading into it.
- When you come to a decision node, pick the best alternative. Note that there are no probabilities associated with the alternatives.

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[1] This is the case, also, with Monte Carlo simulation. New tools are emerging that make optimization more efficient and automatic.

[2] Unless we are minimizing expected value cost.

Sometimes, costs and benefits are incurred as a result of decisions and chance events along the tree. There are two options for placing these values on the decision tree:

- Incorporate all costs and benefits into the terminal node values; or,
- Place values on the branches as incurred; these are then recognized when back-solving the tree. If a cost is placed along a branch, subtract the cost amount from the EMV as you go by, back-solving from right-to-left. Think of the cost as a “toll” to be paid.

Either method can be used, but it is important to select approach and be consistent. Otherwise one risks dropping or double-counting some values.

Here are some additional notes and cautions:

- Special care is required when the analysis measures value in utility units and costs appear along branches: Any costs or benefits placed along tree branches must be in dollars (utilities and $ do not add). Utilities must be converted to ***certain equivalents*** (CEs, in $) before deducting costs placed along a branch.[3] Then convert certain equivalents back to utility units before evaluating the expected utility of the next chance node to the left..
- Do not discount utilities. Utility is a function of PV outcomes.
- All dollar outcomes (normally net of taxes) must recognize any present value discounting. Usually, discount all cashflows to the date of the next investment decision; this is the as-of date of the analysis.
- Make certain that your tree makes sense, chronologically, for your project.

A problem with decision trees is that they appear simple. Like many endeavors, correctly specifying the problem—in this case, the decision model—is half the solution. Drawing meaningful trees is often challenging. One gets better with practice.



[3] Certain equivalents are described in the Risk Preference section.